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Issued on June 2, 2016
Solutions due: June 14, 2016

CS500

Spring 2016, Assignment #5

PROBLEM 15 (2+4+2+2P) **:**

a) Implement the randomized (integer) *Polynomial Identity Test* from the lecture in `elice`.

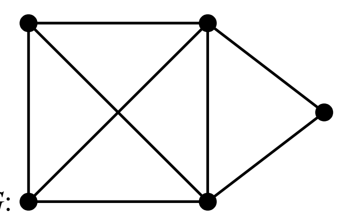
b) Use (a) and Gaussian Elimination, possibly from a library*, to implement in `elice` the randomized algorithm for Perfect Matching in arbitrary graphs with sufficient certainty.

c) Implement in `elice` an algorithm for constructing, given $n, m \in \mathbb{N}$, a graph $G = (V, E)$ with n vertices and m undirected edges uniformly at random.

d) Implement in `elice` an algorithm creating $N = 50$ random graphs, each with $n = 100$ vertices and $m = 350$ edges, and reporting how many of them admit a Perfect Matching; then repeat for $m = 200$.

PROBLEM 16 (1+3+3+1+2P) **:**
Vertex Cover is the following optimization problem: Given an undirected graph $G = (V, E)$, find the least number $k = k(G)$ of vertices $v_1, \ldots, v_k \in V$ such that every edge $e \in E$ is incident to (i.e. has among its two end points) at least one vertex from the set $C = \{v_1, \ldots, v_k\}$. The corresponding decision problem asks whether, given G and ℓ, it holds $k(G) \leq \ell$.

a) Determine $k(G)$ and an optimal Vertex Cover for the following graph G:



b) Establish polynomial-time reducibility of *Vertex Cover* to *Boolean Satisfiability*.
Hint: Consider Boolean variables $x_{v,j}$ for $v \in V$ and $1 \leq j \leq \ell$.

c) Consider the following greedy algorithm, initialized with $C := \{\} =: F$.

> For each edge $e = \{a, b\} \in E$, put e into F and put *both* a, b into C
> and remove from E all edges incident to a or b.

Prove that the resulting set C contains $2|F| \leq 2k(G)$ elements.

d) Construct (a family of) graphs G where the algorithm from (c) produces a vertex cover of size $\geq 2k(G)$.

e) What about the modified heuristic of putting only *one* (arbitrary) of each edge's end points into C?

*Beware of rounding errors!